

Arithmetic Progressions - Exercise 5.4

Exercise 5.4 with independently verified solutions.

CHAPTER

Arithmetic Progressions

RESOURCE

NCERT Exercises

SOURCE

NCERT Mathematics, Reprint 2026-27

PART 1

Verified NCERT questions and solutions

Verified NCERT questions and solutions

Solutions are checked independently and follow the supplied NCERT chapter edition.

QUESTION

Question 1

optional · term number

Which term of 121, 117, 113, ... is the first negative term?

Solution

Board-exam working:

$$a_n = a + (n - 1)d$$

Here $a=121$ and $d=117-121=-4$.

Thus $a_n=121+(n-1)(-4)=125-4n$.

For $a_n < 0$, $125-4n < 0$, so $125 < 4n$ and $n > 31.25$.

The smallest positive integer satisfying this is $n=32$.

Check the boundary: $a_{31}=125-4 \times 31=1$, while $a_{32}=125-4 \times 32=-3$.

Therefore the 32nd term is the first negative term.

Final answer

32nd term (-3).

QUESTION

Question 2

optional · sum of AP

The sum of 3rd and 7th terms is 6 and their product is 8. Find S_{16} .

Solution

Board-exam working:

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

Let $a_3 = a + 2d$ and $a_7 = a + 6d$.

Their sum is $2a + 8d = 6$, so $a + 4d = 3$.

Therefore $a_3 = 3 - 2d$ and $a_7 = 3 + 2d$.

Their product is 8: $(3 - 2d)(3 + 2d) = 8$.

Thus $9 - 4d^2 = 8$, so $4d^2 = 1$ and $d = \pm 1/2$.

Since $a = 3 - 4d$, for $d = 1/2$ we get $a = 1$

for $d = -1/2$ we get $a = 5$.

$$S_{16} = 16/2[2a + 15d] = 8[2(3 - 4d) + 15d] = 8(6 + 7d) = 48 + 56d.$$

Hence $S_{16} = 76$ when $d = 1/2$, or $S_{16} = 20$ when $d = -1/2$.

Final answer

$S_{16} = 76$ or 20 .

QUESTION

Question 3

optional · application

Ladder rungs are 25 cm apart over 2.5 m and decrease from 45 cm to 25 cm. Find wood length for rungs.

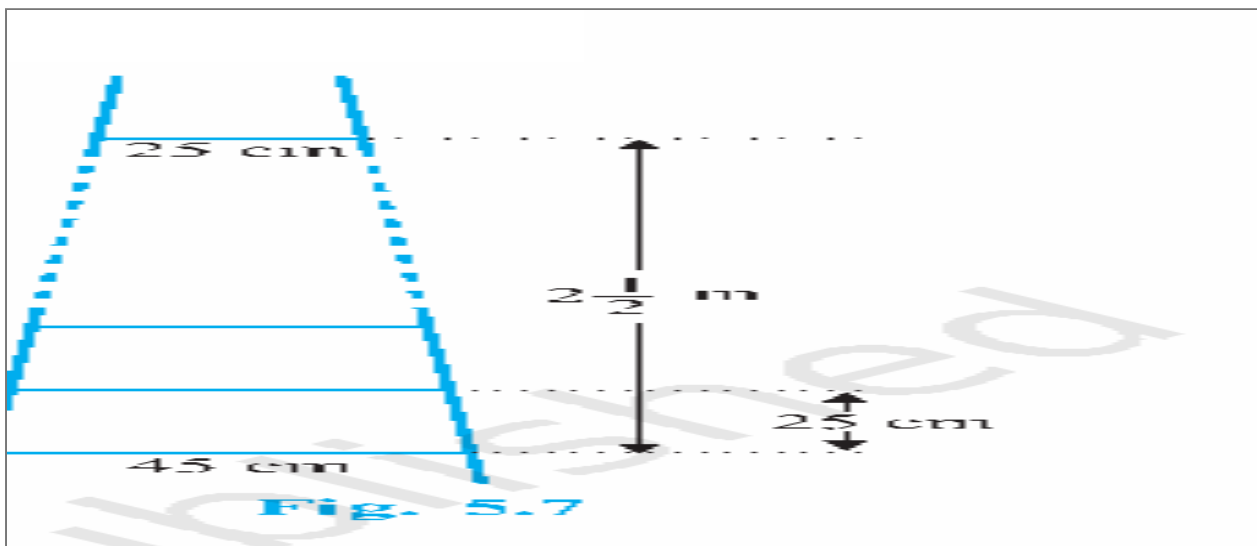


Figure 5.7: ladder dimensions.

Solution

Board-exam working:

Convert 2.5 m to 250 cm.

With 25 cm between consecutive rungs, the number of gaps is $250 \div 25 = 10$.

Hence the number of rungs is $10 + 1 = 11$.

The rung lengths decrease uniformly from 45 cm to 25 cm, so they form an AP with $a = 45, l = 25, n = 11$.

Total wood length = $S_{11} = 11/2(45 + 25) = 11/2 \times 70 = 11 \times 35 = 385$ cm = 3.85 m.

Final answer

385 cm (3.85 m).

QUESTION

Question 4

optional · application

Houses are numbered 1–49. Find x where sum before x equals sum after x .

Solution

Board-exam working:

The sum before house x is $1+2+\dots+(x-1)=x(x-1)/2$.

The total from 1 to 49 is $49 \times 50 / 2 = 1225$.

The sum after x is $1225 - x(x+1)/2$.

Equate the two sums: $x(x-1)/2 = 1225 - x(x+1)/2$.

Multiplying by 2 gives $x(x-1) + x(x+1) = 2450$, so $2x^2 = 2450$ and $x^2 = 1225$.

Thus $x = \pm 35$, but a house number must be positive, so $x = 35$.

Check: $1 + \dots + 34 = 595$ and $36 + \dots + 49 = 595$.

Final answer

$x = 35$.

QUESTION

Question 5

optional · application · sum of AP

A 15-step terrace is 50 m long; each rise is $1/4$ m and tread $1/2$ m. Find concrete volume.

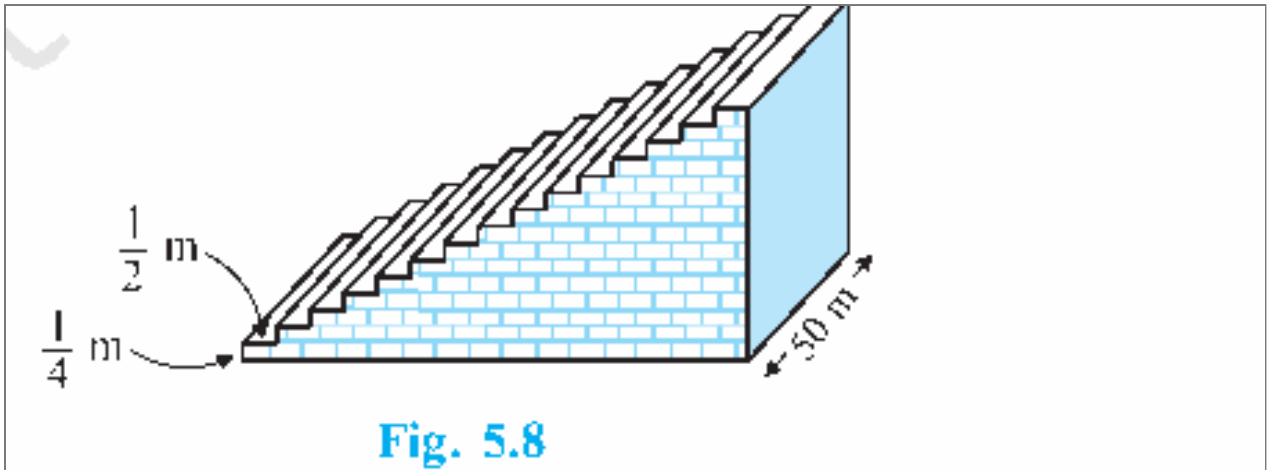


Figure 5.8: concrete terrace dimensions.

Solution

Board-exam working:

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

The first step layer has cross-sectional area $(1/2)(1/4)$, the second contributes twice this amount, and so on up to 15 layers.

With length 50 m, total volume is $50 \times 1/2 \times 1/4 (1+2+\dots+15)$.

The sum $1+2+\dots+15$ is $15/2(1+15)=15/2 \times 16=120$.

Therefore volume $=50 \times 1/2 \times 1/4 \times 120=25/4 \times 120=25 \times 30=750 \text{ m}^3$.

Final answer

750 m^3 .